CUHK Education Journal Vol. 17 No.1 pp.59-72

EXPERT-NOVICE DIFFERENCES IN THE SOLVING OF A BASIC PROBLEM IN CHEMISTRY

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Research into the solving of basic problems for a topic in high school chemistry is reported. Investigations focused on the strategies used and how problems are mentally represented by expert and novice students. All solvers were found to have a repertoire of strategies; the one used depends on the familarity of a problem. Experts mainly employed problem recognition plus a working forwards strategy, acessing a general procedure already available in long-term memory. Novices attempted a means-ends analysis to create a solution procedure; if unsuccessful, they switched to a blind working forwards (groping forwards) strategy. Experts have three successive representations of problems: An initial representation involving a keyword or images of apparatus, an abstract representation capable of qualitatively simulating a solution, and a mathematical representation. Novices sometimes had incomplete abstract representations. Points of comparison are made with problem solving in physics, the predominant domain of research.

Most instruction in science is aimed at achieving two general goals: the acquisition of a body of organized knowledge in a particular domain of science, and the ability to solve problems in that domain. Much research, using the information-processing approach, has already been carried out in physics in both of these goal areas. Far less research has been done in chemistry and the present report deals with one of serveral studies in knowledge acquisition and problem solving in this domain. This report deals with research into the solving of basic quantitative problems in the topic of volumetric analysis. A discussion of the nature of problem solving precedes the report of the research.

The Problem-Solving Process

The meaning of problem solving is viewed differently by different researchers. Some see problem solving as almost synonomous with thinking (e.g., Mayer, 1977) which itself can take on a variety of meanings. Others see problem solving as a particularly complex form of learning which has to be preceded by simpler forms of learning (e.g., Gagne, 1977). Still others view it as an investigative task for which there does not exist an obvious solution at the beginning (Davis, 1966; Mettes, Pilot, Roosnink, & Kramers-Pal, 1981; Gil-Perez & Martinez Terregrosa, 1981). In this third view, problem solving does not include the use of algorithms or other automatic solutions and is very close to the scientific method of solving problems. While a number of researchers seem to adopt this view, there are still conflicting conceptions about what is meant by scientific method and how it is related to problem solving in school science.

However, when viewed from an information-processing framework, problem solving involves finding a solution path leading from the initial problem state (the information in the problem plus knowledge retrieved from memory) to the goal state (the required answer) (Newell & Simon, 1972). The three elements of initial state, goal state, and solution path encompass what Newell and Simon refer to as the problem space. From this viewpoint, problem solving has been described as "a goal-directed sequence of cognitive operations (actions) which a person decides to use that lead from the instructions in a question to the answer or other requested performance" (Anderson, 1980; Chi, Feltovich, & Glaser, 1981). The focal points of investigations into problem solving are the strategies adopted by the solver and conditions such as memory, task perception, or problem representation influencing the problem solving (Kempa & Nicholls, 1983).

No matter what the problem space for a given problem, the same processes are postulated to occur during problem solving. First, the solver forms some initial representation of the problem (Davis, 1984; Champagne, Klopfer, Desena, & Squires, 1981) reflecting a conceptual understanding of the information given in a problem. Second, a strategy is needed to guide the search for a solution through the problem space (Newell & Simon, 1972). This strategy will include the retrieval from memory of additional knowledge relevant to solving the problem. Thirdly, there is a meta-analysis to assess the progress or lack thereof, and to evaluate the success of the solution (Davis, 1984; E. Gagne, 1985; Van Lehn, 1983; Flavell, 1977). Van Lehn (1983) argues that people resort to a meta-analysis when they get stuck during the solution of a problem. Flavell (1977) claims that an increased knowledge about the retrieval and use of relevant knowledge helps the problem solver to minimize errors due to inadequate problem solving techniques; in science problems, this might include a knowledge of formulas and alternative procedures for solving a problem.

Based on the information-processing view of problem solving, the present research was conducted into (a) the search strategies, and (b) the forms of representation used by expert and novice students while solving basic quantitative problems in volumetric analysis.

Problem-Solving Strategies in Science

Search Strategies

Problem solving that is not merely the rote application of algorithms involves a search, using one of a variety of methods, to get from the initial state to the goal state (e.g., Van Lehn, 1983; Newell & Simon, 1972). Because many problems have a large number of possible paths leading to the solution, some kind of strategy is needed to limit this number and to determine which possibility is most likely to lead to success. Strategies to limit the search include working backwards from the desired goal by setting subgoals, until the information given in the problem statement is reached. This strategy seems more suited to the solving of mathematics problems - especially gemoetry proofs - than for science problems (Anderson, 1980).

A powerful form of working backwards is called means-ends analysis. This strategy has been reported extensively in the solving of physics problems (e.g., Larkin, 1983; Larkin, McDermott, Simon, & Simon, 1980; Simon & Simon, 1978). Means-ends analysis involves several steps: (a) finding the difference between the goal and the current information in the problem; (b) finding an operation that helps to reduce this difference, such as the use of a formula or equation; (c) carrying out this operation; and (d) repeating steps a - c recursively with a series of subgoals until a solution path has been found. Step d is necessary because the first operation/formula chosen may include a variable for which no data is given in the problem. A subgoal is therefore set up to determine the value for this variable; this will involve another operation/formula, and so the process continues recursively until an operation can be executed using data that is given. Means-ends analysis differs from a working backwards strategy; though both consider the goal first and operations to achieve that goal, means-ends analysis considers the differences between the goal and the current information (state) whereas pure working backwards does not (Anderson, 1980).

Another way to limit search, which is not as powerful as means-analysis, is by using the working forwards strategy. This starts with the current situation and performs operations to transform it until the goal is reached. Means-ends analysis is a more powerful strategy because it requires more knowledge of operations that reduce differences between the current state and the goal. Working forwards is efficient, saves time and is suitable if one knows how to arrive at the answer (Kramers-Pal, Lambrechts, & Wolff, 1983). For unfamiliar problems, working forwards may be less desirable because many solution paths lead away from the goal to dead ends (Kramers-Pal et al., 1983; Anderson, 1980).

Expert-Novice Difference

A number of interesting results have been obtained in the studies of problem solving by experts and novices, mainly in the domain of physics (e.g., Bhaskar & Simon, 1977; Chi, Glaser, & Rees, 1982; Larkin & Reif, 1981; Larkin, McDermott, Simon, & Simon, 1980; Greeno, 1983). A major finding is that experts use a strategy quite different from that employed by novices. Larkin et al. (1980), Chi et al. (1981), and Greeno (1983) have reported that experts use a working forwards strategy, proceding from the quantities given in a problem to the desired goal and that this is done qualitatively before equations are used for a quantitative solution. By contrast, novices begin with a means-ends analysis strategy, searching for an equation that will link the goal to the data in the problem. The qualitative solution by experts is low in detail and involves physics principles; it is utilized by students to guide them in the writing of appropriate mathematical relationships. Novices however do not do a qualitative simulation but just seek to solve the problem using mathematical formulas to represent the principles.

Problem Representation: Expert-Novice Differences

Problem representation has featured largegly in the work of serveral researchers (e.g., Larkin, 1983; Larkin & Reif, 1981; Chi, Feltovich, & Glaser, 1981). It turns out that experts differ a great deal in how they represent problems in physics. One technique for studying problem representation is to ask people regarded as experts and novices in a domain to think aloud while solving problems and then to carry out an analysis of their solution protocols. This method was used in an early study by Larkin and Reif (1981) who got a physics professor (whom they labelled the expert) and an undergraduate physics student who had just completed a first course in mechanics (whom the researchers defined as the novice) to solve several problems in mechanics. The novice first constructed a diagram from the data in the problem. Then he constructed a mathematical representation of the problem. He identified equations (such as Newton's equations of motion) which he then solved mathematically.

By contrast, the expert, after constructing a diagrammatic representation, did not immediately construct a mathematical representation of the problem. Instead he selected key physics principles to build up a qualitative description of the problem solution. Only when this qualitative description was complete did the expert formulate a mathematical representation of the problem to derive the necessary numerical solution.

In a later study, Larkin (1983) elaborates on the representations used by experts and novices in mechanics. Novices have what she calls a naive representation of a problem. This is an internal representation of the problem that contains entities corresponding to the real-world objects referred to in a problem, such as blocks, pulleys, and springs. This representation enables the novice to simulate how the objects would behave in a real situation such as the movement of a block down an inclined plane, but provides little guidance in selecting principles for application. Without an understanding of the physics principles, the novices used mathematical equations combined with a means-ends analysis to reach the desired goal. Experts, in addition to being capable of having a naive representation, construct what Larkin calls a physical representation. This involves entities that correspond to abstract physics concepts (e.g., force, energy, momentum). Therefore the expert has a second representation, one that is based on an understanding of physics principles and which enables the solver to carry out a qualitative simulation to determine how the problem can be solved.

Thus in solving a mechanics problem, an expert may use up to three successive representations: (a) A naive representation taking the form of a diagram, (b) A physical representation involving physics principles, and (c) A mathematical representation consisting of a set of equations to be solved to get the answer. The novice, by contrast, uses just the naive and the mathematical representations.

Another approach for studying problem representation in experts and novices was used by Chi, Feltovich, and Glaser (1981). They asked novices (university students with one course in physics) and experts (Ph.D. physicists) to categorize physics problems. Some of the problems had similar surface features such as diagrams (e.g., inclined planes) but involved different physics principles. Other problems used the same principle (e.g., the conservation of energy), but had different diagrams (such as an inclined plane in one and an oscillating spring in another). Chi et al. found that experts grouped problems according to a shared principle independent of the surface features of the problems. This would agree with the findings of Larkin (1983) that the expert's solution is guided by an abstract representation of the problem related to the laws of physics. On the other hand, Chi et al. (1981) found that the novices grouped problems largely according to the diagrams accompanying the problem statements. Together with the Larkin (1983) study, these findings suggest that, at least in physics, novices represent the more superficial aspects of a problem than do experts.

Little explicit work appears in the literature on expert and novice problem representation in other science domains such as chemistry and biology. A study by Yarroch (1985) investigated the ability of high school students to balance simple chemical equations, and how the students represented these equations. All students were able to successfully balance the four equations presented to them, and all used an abstract representation based on symbols for chemical substances and laws relating to the balancing of equations. Twelve of the 14 students in the study were able to draw a diagrammatic representation of the balanced equations when asked, though it was not reported whether the students actually used this representation when doing the problems. The differences that did emerge in the problem solving of these students was not primarily one of the kinds of representations used, but in the accuracy of those representations. Only five of the students were able to draw diagrams that were consistent with the notion of the balanced equation; the same five also had a better understanding of the laws of chemistry underlying the writing and balancing of chemical equations.

Problem-Solving Strategies for Basic Volumetric Analysis Problems

Method

Two high school chemistry classes from one school in Hong Kong provided the students for the study. The classes were Form 6 with 28 students and Form 5 with 39 students. To select the expert and novice students, all students in the two classes were given a conventional problem-solving test in volumetric analysis. The problems were similar to those that they would have met in class. Students who made no procedural errors were classified as experts while those with most problems solved incorrectly were labelled as novices.

Six experts were selected at random from Form 6 and six novices were drawn from the Form 5 class. To ascertain (a) the problem-solving strategies and (b) the mental representations used during problem solving, two interview methods were employed. The first was the think-aloud procedure, in which students were instructed to talk out loud while solving problems. The second procedure involved the use of probing questions after a problem had been solved to elicit more information about the mental processes used while problem solving. All interviews were conducted in English.

The basic problem below was one used during the interviews:

A solution contains 1.1 g of sodium nitrate in 250 cm^3 of solution. What is the concentration of this solution?

The atomic masses needed to solve the problem were provided on a separate data sheet.

Every interview was audio-taped and transcripts made to provide protocols of the sessions. The protocols were then analyzed in order to infer the strategies and representations used. Protocol analysis is necessary as it is almost impossible to draw adequate inferences during the actual interviews. Whenever the inferences could not fully account for the responses of the students, further interviews based on these inferences were carried out and the findings refined.

Results and Discussion

The results reported here deal only with the salient aspects of the research findings. Because of the length and number of student protocols, many specific references and quotations have had to be omitted.

In sumary, the major finding is that there are many similarities in the problem solving of both expert and novice students. What differs are the conditions under which specific strategies and representations are manifested, *particularly whether the* problem is seen as familiar or unfamiliar.

Problem-Solving Strategies

In solving a problem, it is necessary to have a solution path leading from the information in a problem to the goal. For basic problems such as that above, both experts and novices employ a variety of strategies. These include the powerful means-ends analysis, working forwards, and occasionally a combination of forward and backward reasoning.

Strategies for Solving Basic Problems: Experts

When solving basic problems, experts tend to use the working-forwards strategy. The problem is perceived as familiar and the solution procedures have been used many times before. The path to the goal is found either before the problem is executed or simultaneously with the executed solution.

Consider the protocol and written solution for our basic volumetric analysis problem, which is shown in Table 1. The written procedure consists of two steps: a) Calculation of the number of moles of sodium nitrate, and (b) calculation of the concentration (molarity) of the solution. The expert student, after reading the problem, immediately began to talk and to execute the solution. In lines 1-3 of his portocol, he thinks about the number of moles of sodium nitrate and completes the necessary calculation. From lines 4-6, calculation of the molarity is done at the same time as he thinks about the step.

A second expert student also began immediately to solve the problem. However, he simulated the complete problem solution first before putting pen to paper as indicated by his protocol:

Firstly I find the number of moles of the sodium nitrate, and I know the number of moles of sodium nitrate, and I also know the volume of it, so I know the number of moles equals volume times molarity, so I can find the concertration of the solution.

All told, 5 of the 6 expert students solved the problem using the same strategy and produced very similar protocols.

In solving this problem, these 5 experts first thought of moles of solute, then molarity of the solution. Proceeding in this manner suggests that a working-forwards strategy is used. However, the rapidity with which the experts solved the probelm indicates that little, if any, search for a solution is necessary. The problem and its solution appeared to be immediately recognized by the experts.When asked if they recognized the problem, all the experts gave responses similar to that below by one student:

Yes, immediately! When I look at the question I think I know how to do it...... Because it is common. We've done it many times before.

The key to the experts' strategy seems to lie in this familiarity. Experts look at the problem and can recognize the kind of problem and the procedure that will apply. The general procedure, which must already be available in long-term memory, is instantiated with data from the current problem. To use terms such as "problem solving" and "working forwards search strategy" may be misleading. The problem has not been genuinely solved - the solution procedure is already available and does not have to be created.Also, no search is involved - the experts, while still working forwards, merely follows a well-worn path and does not consciously have to make decisions at each step as to the next formula to be used. This finding tends to confirm the suggestion by Kramers-Pal, Lambrechts, and Wolff (1983) that for an expert, a problem is no real problem at all but a standard problem for which a problem-solving sequence is applied almost automatically.

Table 1
The Solution by Expert Students for a Basic Problem in Volumetric Analysis together with the Protocol of One Student

Problem

A solution contains 1.1g of sodium nitrate in $250cm^3$ of solution. What is the concentration of this solution? (Atomic masses: sodium 23, nitrogen 14, oxygen, 16.)

Written Solution

No. of moles of sodium nitrate = mass / molar mass

$$= \frac{1.1}{23+14+16\times 3}$$

$$=0.129$$

Molarity = moles / volume

$$= \frac{0.129}{0.25}$$

$$=0.052M$$

Protocol of One Expert Student

1. Calculate the number of moles of sodium nitrate in 1.1g.
2. ... equals 1.1/(23+14+16×3) equals .... 0.129. [S uses
3. calculator and writes this while talking]
4. Second... because molarity equals number of moles over volume, so
5. molarity equals 0.129 / 0.25 equals.... 0.052 M. [S again calculates,

6. writes while talking]
7. Therefore the concentration of the solution is 0.052 M.

The problem recognition and working-forwards strategy is therefore used whenever a problem is familiar and the solution apparent. However, if the problem is perceived as being more difficult, experts can switch to the more powerful means-ends analysis strategy. With this, the goal statement of the problem is sought first, and a formula is selected which relates the goal variable to other variables given in the problem statement. The sixth expert student perceived our problem as more difficult and after reading the question found the solution path using a means-ends analysis. The student's think-aloud portocol is as follows:

I'll first think of concertration is the answer I must find and then...

I will use the equation: number of moles = molarity × volume, where molarity is the concentration we are going to find.

To find out how may moles of sodium nitrate, I must know the atomic masses so that I can calculate ... mole number of sodium nitrate.

The student first identifies the goal of concentration, and then a formula linking the goal with the data (i.e., moles = molarity × volume). A subgoal is set up to find the number of moles as the value of this variable is not given. Although not stated in the protocol, the formula is: moles = mass / molar mass. Mass is given in the problem but molar mass is now an unknown. The further subgoal of molar mass is obtained with the given atomic masses. This strategy therefore suggests a means-ends analysis.

With means-ends analysis, the procedural steps are created in the reverse order to the procedure that is actually executed on paper. Execution of a problem is always forwards, however. Therefore, merely looking at the written procedure gives little indication of the mental strategy employed by a student.

Strategies for Solving Basic Problems: Novices

Basic volumetric analysis problems do not have the familiarity or recognizable solution paths for novices as they do for experts. The novices in the study were found to use means-ends analysis when solving these problems. In contrast to experts, novices were slower, used many incorrect formulas, and during the think-aloud sessions there were frequent pauses. Follow-up questioning was used to elicit the thinking that took place during these pauses.

The protocol and written solution for th basic problem under discussion is shown in Table 2 for one novice student. The protocol suggests that a means-ends analysis has been used but further questions were asked regarding the order in which the steps were thought of:

Table 2
The Think-aloud Protocol and Written Solution of a Novice Student for the Basic Problem

Protocol of One Novice Student

[S reads the problem, then a long pause.]
1. At first we calculate the....
2. We use the formula about... moles equals MV [S writes down this formula]
4. And find the concentration of M.
5. [Long pause] Number of moles of sodium nitrate equals... [S writes down
6. no. of moles of sodium nitrate = mass × molar mass]
7. [Long pause] No, it's wrong. Equals mass / molar mass. [S completes the
8. calculation of moles of sodium nitrate]
9. The number of moles of sodium nitrate is 0.013.
10. Then I use the information...

equation: moles equals MV.

11. Concentration means molarity. Concentration of the solution equals mole/volume.
12. Volume in dm³. [S completes the step for concentration]
13. The concentration of this solution eqals 0.052

Written Solution of the Novice Student

mole=M X V (lines 1-4)

no. of mole of sodium nitrate

=mass × molar mass (lines 5-6)

=1.1g × (23+14+16×3)

=1.1×85

=[sic]

$= \frac{\text{mass}}{\text{molar mass}}$ (lines 7-9)

$= \frac{1.1}{23+14+16\times3}$

=0.013

mole=M×V

$M = \frac{\text{mole}}{V}$ (lines 10)

(lines 10-12)

$= \frac{0.013}{0.25\ dm^3}$

=0.052M (lines 13)

Interviewer (I): What did you think of first when you did this problem?

Student (S): Moles=MV.

I: Why this first?

S: Because I want concentration; concentration is molarity. The question gives volume and has mass, so I think this equation may be true.

I: But you don't have the number of moles.

S: So we use the equation mass/ molar mass [to get moles].

Choosing a formula linking the goal variable (concentration) with given variables, then working backwards recursively to obtain a subgoal (number of moles) confirms the use of means-ends analysis. The strategy used and the procedure derived are thus the same as for the expert student who used means-ends analysis.

If a means-ends analysis cannot be initated because the novice is not able to think of a formula linking the goal to the data, the student switches to a strategy of working forwards. However, this working forwards is not the same as for an expert, being much more of a groping-in-the-dark approach. I have named this approach the groping forwards strategy. Using formulas that he or she is less sure of, the novice proceeds one step at a time, hoping to be able to use the information derived from one step to derive another, until the goal is reached. If this strategy is unsuccessful, the problem solving is terminated. Notice that in the groping forwards strategy, the order of reasoning and the order of the written solution are the same.

One novice student switched to this groping forwards strategy when she could not think of a formula for concentration in the problem as indicated in the following protocol of the follow-up interview after the problem solving had been completed:

I: What was the first thing you thought of when solving this problem?

S: The answer to find.

I: And then?

S: I have forgotten the formula for concentration.

I: What did you think of then?

S: Sodium nitrate. It is the main thing... If we find the number of moles of sodium nitrate, we can find out many things, such as molarity.

I: Are you thinking of molarity when you think of number of moles?

S: No!

I: Are you sure you will use it [moles] later?

S: No. I hope I can.

I: What do you do next?

S: A formula about using number of moles. One with molarity.

Thus the order: moles of sodium nitrate → molarity, shows a working forwards strategy is used. The groping forwards strategy is less powerful than a means-ends analysis as many solution paths are possible depending on the formulas (often incorrect) used at each step of the procedure. Most of these paths will be erroneous and the novice is not sure of the answer even if it is correct. In the above example, the student had in fact used the correct procedure to calculate molarity, but believed it be be wrong because she did not initially realize that molarity is concentration. Had she done so, a means-ends analysis could have been used to solve the problem.

Both the novices and the experts in the study then appear to have a variety of alternative general strategies available. The strategy actually employed depends on the familiarity of the problem and whether or not an impasse is reached during the solution of the problem. This suggests that the students are also carrying out a meta-analysis to facilitate the choice of a problem-solving strategy. The availability of different strategies contrasts with the problem-solving research in physics, and particularly that of Larkin. Her experts seemed to use a working forwards strategy exclusively. The present study indicates that even experts (at least among high school students) will also resort to a means-ends analysis when necessary. It is hypothesized that teachers, university professors (the experts in the research of Larkin and others), or any experts will also do the same for problems that are less familiar.

Problem Representation

In summary, when students solve basic volumetric analysis problems, up to three representations are employed, as follows:

1. An initial representation which may be concrete or abstract.
2. This initial representation may be followed by the construction of a qualitative representation of the problem which is able to qualitatively simulate the solution path.
3. A mathematical representation which uses appropriate formulas and numerical data. This representation provides the observable written procedure for the problem.

The Initial Representation

For the given problem, all students set up some initial representation. A word such as molarity or concentration is used to represent the kind of problem and to guide subsequent thinking. All students who were aked, mentioned the use of a keyword. The initial representation may also be concrete, involving mental images of laboratory apparatus or procedures. Students do not use either a concrete or a keyword representation exclusively. A keyword representation tends to be used immediately for a problem that is familiar, whereas an initial concrete representation is used to understand more readily a problem regarded as less familiar or difficult. For a concrete representation, students sometimes draw a rough diagram of laborarory apparatus on their answer sheets. One novice comments on the use of mental images:

I: *Do you think of an experiment with apparatus or just formulas and data when you do this problem?*
S: *An experiment..... I think of this.... [S draws a diagram of apparatus]...*
I: *Does thinking of apparatus help you to solve the problem easier?*
S: *No.*

Most students who thought of mental images said that the images were not important nor useful in helping them in solving the problem. The still had to think of the abstract terms or mathematical formulas in the problem, that is, to set up subsequent representations. This initial representation for volumetric analysis problems appears to be much less elaborate than those constructed for force and motion problems in physics mentioned earlier. Many physics students, especially novices, were found to set up initial representations that enabled the simulation of the behaviour of the objects given in the problem. However, these elaborate initial representations still provided little guidance in setting up subsequent representations necessary to solve the physics problems.

Qualitative Simulation of Problem Solutions

Following the construction of the initial representation, all expert students and many novices were able to construct a representation capable of simulating the abstract solution procedure, a point already made when discussing search strategies. During this construction, a small number of key variables (e.g., moles, molarity), are linked together to generate an outline procedure for solving the problem. Although data and formulas may be referred to while the representation is being constructed, this is done in order to abstract the major entities in the procedure. The procedural details are only called on when the final mathematical solution to the problem is executed. This is reflected in the responses of one expert student when questioned about his simulated abstract representation:

I: *Do you think about the little things when working out the solution?*
S: *No! Just when I start the calculation I think about the details, for example, the mass, what are the atomic masses .... I just find the main things.*
I: *What are the main things in this problem?*
S: *Just how many moles, molarity .... Volume, things like that are not important now.*

Hence, the abstract representation for our basic problem is:

<u>Step 1.</u> (Find the number of) moles.
<u>Step 2.</u> (Find the) molarity.

This abstract representation can be used for simulating the problem solution, and for providing answers to questions about the procedure. The representation is parsimonious; only two entities are included, thus reducing the load on working memory. This qualitative solution path is now used to guide the numerical solution of the problem. Substeps will need to be included in order to complete the calculation in each step. These are the details that are filled in during execution.

With experts, for whom the problem is familiar, the general abstract representation of the solution is probably stored in long-term memory. It is accessed, by the working forwards strategy, when needed. When the problem is less familiar, as is the case with novices, the representation is created during application of the means-ends analysis. The solver works backwards form the goal to the given information, retaining key variables in working memory. For the basic problem only two variables are needed to qualitatively simulate the solution. Thus working memory is not overloaded. These findings contrast with those of Larkin who claims that novices do not from a qualitative representation of a problem while using a means-ends analysis strategy. This could be related to the more complex, harder problems she used in her studies, often involving multi-step procedures. The present research with basic (chemistry) problems suggests that novices are capable of qualitatively simulating a solution.

For novice students using a groping forwards strategy however, the lack of clarity in explaining or answering questions about a problem procedure suggests that they do not carry out a complete qualitative simulation of the solution. Working blindly from step to step, they appear unable to isolate and link the key variables into a coherent qualitative solution.

The Mathematical Representation

Once a qualitative representation of the problem is set up, a mathematical representation is constructed. The key variables in the abstract representation are used to guide the selection of appropriate formulas into which numerical data are substituted. The result is the quantitative procedures and numerical answers shown in Table 1 and Table 2. The details of the solution (such as molar masses and volumes

Table 3
A summary of Expert and Novice Student Problem Solving for Basic Problems in Volumetric Analysis

Experts	Novices
Strategy for finding procedures	
* Variety of strategies.	* Variety of strategies.
* Mainly a problem recognition and working fowards strategy.	* Mainly a means-ends analysis strategy.
* No search needed — known procedures accessed from long-term memory.	* Creative search for a solution procedure involved.
	* If a means-ends analysis is unsuccessful, switch to a groping forwards search strategy.
Problem representation	
* Entities mainly abstract.	* Entities mainly abstract.
* Sometimes use an additional concrete initial representation.	* Use an initial concrete reprentation for less familiar problems.
* Abstract representation and qualitative simulation before a quantitative, mathematical representation.	* Qualitative representation and simulation only if a means-ends analysis is used; otherwise formula driven.

and how to calculate them), not present in the parsimonious qualitative representation, are filled in at this stage. This mathematical representation does not supplant the qualitative procedure, which is still present to enable to student to do any further simulation or explanation of the procedure that is required.

If the full qualitative representation to guide the mathematical solution is lacking, as when novices use a groping forwards strategy, the execution is formula driven. From one step to the next, a formula is selected and executed until a numerical answer for the goal term is arrived at.

Summary

A summary of expert and novice student problem solving for basic problems in volumetric analysis is shown in Table 3. By means of a strategy for deriving the solution path together with evolving representations of the problems, the solver builds up a complex mental model of the problem and its solution.

Expert students employ a strategy of problem recognition coupled with working forwards to obtain the procedure. No search for a solution path is needed. Problems are familiar and known general procedures are available in long-term memory. The expert usually begins with a keyword representation to identify the problem. Occasionally a concrete representation consisting of mental images of laboratory apparatus or procedures is also constructed though this representation is not important for subsequent solving of the problem. Accessing the stored general procedure, and using a working forwards strategy, the expert constructs an abstract representation. This representation is capable of qualitatively simulating the solution and is parsimonious, consisting of a small number of key variables linked together to give a qualitative solution. The qualitative representation then guides the selection of appropriate formulas for application to build up a mathematical representation to enable the quantitative solution of the problem to be obtained.

Novice students, and occasionally experts, have difficulty in recognizing how the problem should be solved as no procedure is available in memory. Following the same kind of initial representation as the expert, the novice employs a means-ends analysis strategy to create a solution path. If this strategy is successful, an abstract qualitative representation is constructed, followed by the mathematical representation and numerical solution. Sometimes the means-ends analysis is unsuccessful. In the case, the novice switches to a blind working forwards strategy (labelled the groping forwards strategy) using any formulas (usually erroneous) and data in the hope that the steps made will lead to a solution. Little qualitative simulation of the problem occurs and seldom is the problem solved correctly or even completed.

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