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Quantitative Problem Solving in Science: Cognitive Factors and Directions for Practice

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This paper presents an overview of how a number of cognitive factors are involved in the solving of quantitative problems by secondary school students categorized as skilled and less skilled. Illustrations are given for the solving of basic problems as well as more complex composite problems in chemistry. The factors discussed are those of problem recognition, search strategy, problem representation, linkage, cognitive load and the role of understanding versus algorithms. Based on the discussion, directions for the enhancement of instruction in problem solving are suggested. These include the key role of instruction for conceptual understanding, encouragement of qualitative thinking by problem solvers, the setting of goal-free problems and practice with basic problems.

Introduction

An area of difficulty for many students in secondary school science is the solving of quantitative problems. The past two decades have seen a great deal of work in the study of problem solving and there is a growing consensus about the kinds of mental processes involved in human problem solving, the cognitive factors involved and the kinds of difficulties problem solvers have. The dominant perspective for problem solving is the information-processing approach (Newell & Simon, 1972), the goals of which are to provide precise descriptions of the mechanisms of human problem solving, the causes of errors, differences between skilled and less skilled performance and, from an educational standpoint, the hope of improving instruction. Such an approach has been used by Reif (1981), Larkin (1983) and others to obtain explicit models of problem solving in physics. According to the approach, two basic cognitive factors are involved. First, there is the construction by the solver of an adequate representation of the problem reflecting an understanding of the information given in the problem statement. Second, a strategy is used to guide the search for a solution from the information and data given to the required answer. In addition to these two factors, other factors have been identified as being important in problem solving. They include problem recognition, linkage, cognitive load and prior knowledge (e.g., Ayres, 1993; Bunce, Gabel, & Samuel, 1991; Lee, 1985; Lee, Goh, Chia, & Chin, 1996; Lythcott, 1990; Sweller, 1988; White, 1988).

The purpose of this paper is twofold: (a) To present an overview of a number of cognitive factors involved in quantitative problem solving in science and how these factors mediate the performance of skilled and less skilled problem solvers, and (b) to suggest some directions for classroom instruction to facilitate more effective problem solving. For illustrative purposes, sample quantitative problems are taken from the chemistry topic of volumetric analysis though reference is made to problem solving in other subject areas. The skilled and less skilled problem solvers are from chemistry classes in one secondary school in Hong Kong who have taken an introductory course in volumetric analysis. The skilled students were taken to be those who made no procedural errors in conventional problem solving tests (i.e., neglecting arithmetic slips), while the less skilled students were those whose problem-solving procedures were largely erroneous or incomplete. A detailed analysis of the quantitative problem solving providing empirical evidence for the findings presented here is discussed elsewhere (Heyworth, in press-a, in press-b).

A Description of the Problems

In terms of complexity, quantitative problems in subjects such as chemistry and physics are of two kinds. First there are basic problems which require a small number of steps, often only one or two, to solve them. Then there are composite problems which require multi-step procedures to solve them and which often consist of a number of the basic problems linked together. Students have to identify these component subproblems and devise a procedure to solve the whole problem.

Here is an example of a composite problem from the topic of volumetric analysis:

> 125 cm^3 of a solution of sodium hydroxide contains 4 g of solute. In a titration, a student finds that 50 cm^3 of this sodium hydroxide solution completely neutralizes 20 cm^3 of a hydrochloric acid solution. Find the volume of this hydrochloric acid the student must dilute to make exactly 1 dm^3 of 1.2 M hydrochloric acid.
> [Relative atomic masses: Na 23, O 16, H 1]

This composite problem consists of three parts each corresponding to a basic problem. These parts, and how they would appear when written as basic problems are as follows:

1. A solution of sodium hydroxide for which the concentration needs to be found.
   The basic problem is of the type:
   125 cm^3 of a solution of sodium hydroxide contains 4 g of solute. What is the molarity (concentration) of the solution?
2. A titration for the neutralization of sodium hydroxide solution and hydrochloric acid.
   The basic problem is of the type:
   In a titration, a student finds that 50 cm^3 of [a 0.8 M] sodium hydroxide solution completely neutralizes 20 cm^3 of a hydrochloric acid solution. What is the concentration of the hydrochloric acid?
3. The dilution of the hydrochloric acid solution.
   The basic problem is of the type:
   Find the volume of 2 M hydrochloric acid that must be diluted to make exactly 1 dm^3 of 1.2 M hydrochloric acid.

The standard procedures for solving the basic problems involve just a few steps. For example, Basic Problem 1 requires two steps: (a) the calculation of the number of moles of solute (sodium hydroxide), and (b) using the value for number of moles to determine the molarity of the solution. By linking the procedures for the three basic problems, a procedure for solving the longer composite problem can be derived. An important difference between the composite problem and the basic problems is that the latter include goal statements which are absent in the composite problem. As we will see, this difference has important consequences for the solving of such problems by less skilled students.

Cognitive Factors in Science Problem Solving: An Overview

Problem Recognition

The first step in problem solving is being able to recognize the type of problem being solved. The research literature has shown problem recognition to be an important determinant of success in problem solving and that it is closely related to prior experience in solving similar problems (e.g., Chi, Feltovich, & Glaser, 1981; Frazer & Sleet, 1984; Gabel & Bunce, 1994; Lee, 1985; Lee et al., 1996). To facilitate problem recognition, skilled and less skilled problem solvers alike concentrate on key words in the problem statement (e.g., Chi et al., 1981). For the volumetric problems above, examples of key words are "molarity," "titration" and "dilution." For basic problems, the key words serve to identify the type of problem. For composite problems, they assist in the decomposition of the problems into familiar, recognizable components (Bunce et al., 1991; Heyworth, in press-a, in press-b).

In the studies referred to above, skilled students were found to be good at problem recognition and at decomposing composite problems. Less skilled students are usually able to recognize basic problems, based on prior experience, though they may not always be able to solve them correctly. However, they are less successful at interpreting problem statements in composite problems and in decomposing these problems into component problems. A major reason for the difficulty in basic problem recognition may be that subgoals (e.g., "Calculate the concentration of the solution.") are not provided as specific cues in composite problems to help in identifying the kinds of basic problems. Without clearly recognizable basic problems, a composite problem becomes just one big problem and tends to be treated by less skilled problem solvers in the same way as basic problems when attempting to derive solutions. Difficulty in recognition affects other cognitive factors such as the strategy employed in creating a solution procedure and how the procedure is represented as is discussed below.

Strategies

Having recognized a problem or its components, it is then necessary to use some strategy to obtain a solution path leading from the information given in the problem statement to the goal (the required answer). Based on analyses of student think-aloud protocols of problem solving, two basic strategies have been observed in a number of subjects including physics, chemistry and mathematics, viz., the "working forwards strategy" and the "means-ends analysis strategy" (e.g., Ayres, 1993; Larkin. 1983; Owen & Sweller, 1985; Sweller, 1988). The choice of a strategy appears to be related to the familiarity of a problem. For familiar problems, with which students have a lot of experience, working forwards is the main strategy used whereas means-ends analysis tends to be used when a problem is less familiar or unfamiliar.

Working forwards has been found to be the dominant strategy used by skilled problem solvers. The solver begins with the current information in the problem statement and works forwards, performing operations to transform this until the goal is reached. For example, Basic Problem 1 above is solved by skilled students in two steps as follows:

Step 1: Calculate the number of moles of sodium hydroxide (solute).
Step 2: Calculate the molarity of the solution.

For basic or simple problems, which are very familiar to skilled problem solvers, the general procedures have been well practised and are encoded in long-term memory from where they can be accessed and used to solve future problems in a routine working forwards manner. For more complex problems, skilled problem solvers also tend to use the working forwards strategy. In studies by Larkin (1983) of problem solving in physics, skilled problem solvers (university professors) seemed to use a working forwards strategy exclusively when solving such problems. Working forwards can be an efficient strategy as it saves time if the problem is familiar and the solver knows how to arrive at the answer (Kramers-Pals, Lambrechts, & Wolff, 1983).

The means-ends analysis strategy is commonly, though not exclusively, used by less skilled problem solvers and tends to be employed when a problem is new or unfamiliar and a procedure for solving it is not readily available. Means-ends analysis is a form of backward reasoning and involves (a) identifying the goal statement, (b) finding the difference between the goal and the current information, (c) finding an operation that will reduce this difference (such as using a formula or equation), (d) attempting to carry out this operation, and if this is not possible then (e) repeating steps (b) to (d) recursively with a series of subgoals until a solution path is found. The procedure generated must be held in working memory in the reverse order to that which will be used to obtain the written solution. Thus for Basic Problem 1, less skilled students who were able to solve it correctly were observed, during think-aloud sessions, to follow this reasoning. They generated the following steps:

Goal: Find the molarity of the solution.
Subgoal 1: To find molarity, calculate the moles of solute in the solution.
Subgoal 2: To find moles of solute, use mass (given) and calculate molar mass (from given data).

The search process is now complete as a procedure linking the goal to the given information has been generated. To obtain the written solution, the steps of this procedure are reversed. Means-ends analysis has been reported for the solving of problems in other subjects such as physics and mathematics, and with problem solvers ranging from primary school pupils to university students (e.g., Larkin, 1983; Sweller, 1988; Sweller, Mawer, & Ward, 1983). The less skilled Hong Kong students attempted means-ends analysis for both basic and composite problems alike though with mixed success. In fact, for the composite problem, none was able to solve it correctly. Although the skilled Hong Kong students used a working forwards strategy, it is probable that when these problems were first met, means-ends analysis would have been used to create procedures and that with time and familiarity, they switched to working forwards.

Less skilled students, however, often cannot complete a means-ends analysis as they are unable to think of a formula linking the goal statement to other data, such as a formula linking molarity to given data in Basic Problem 1. In this situation the students switch from means-ends analysis to a working-forwards strategy. However, this working forwards is not the same as for the skilled student but is much more of a groping-in-the-dark approach. Thus I have named this approach the "groping forwards strategy." The less skilled student begins by taking numerical values from the problem statement and substituting them into *any* formula that can be recalled, even if this is erroneous. This process continues using given or derived data until a value for the goal variable is reached. Thus one less skilled student created the following procedure for Basic Problem 1 using the groping forwards strategy:

Step 1: Calculate molar mass of sodium hydroxide (done correctly using the formula: molar mass = Σ relative atomic masses).
Step 2: Calculate molarity (using an erroneous formula: molarity = molar mass/volume).

As with all working forwards strategies, the order of reasoning for the groping forwards strategy and the order of the written procedure are the same. If the groping forwards strategy fails to produce an answer, the problem solving is terminated. This occurred frequently in the solving of the composite problem by the less skilled Hong Kong students.

Problem Representation

While a problem is being solved, studies have shown that the representation of the problem changes and that for skilled and less skilled problem solvers these changes are qualitatively different (e.g., Chi et al., 1981; Coleman & Shore, 1991; Larkin, 1983).

Up to three representations are employed. These are:

1. An initial representation which may be concrete or abstract.
2. A qualitative representation which provides an outline of the solution procedure.
3. A mathematical representation which corresponds to the written solution to the problem.

During the problem recognition stage, both skilled and less skilled problem solvers set up some initial representation based on key words in the problem statement. The information is often closely tied to real, familiar objects (e.g., Larkin, 1983; Slotta, Chi, & Joram, 1995) which in the case of the above chemistry problems are images of laboratory apparatus or procedures. Following the setting up of this representation, skilled and less skilled problem solvers differ. Skilled problem solvers convert this initial representation into one containing abstract entities from formal science which enables a *qualitative* solution procedure to be constructed. The qualitative representation is a key element in effective problem solving. It is a high level representation containing a small number of steps to give an *outline* of the procedure; details of the procedure are not present though they may be referred to while it is being constructed.

The qualitative procedure is used to set up the mathematical representation by guiding the selection of appropriate formulas resulting in a quantitative procedure and numerical answer. Procedural details, not present in the qualitative procedure, are filled in at this stage. The mathematical procedure does not supplant the qualitative procedure which is still present to enable the problem solver to do any further explanation or simulation of the procedure that is required. Here is an example of the qualitative procedure constructed by one skilled student (while using a working forwards strategy) for the composite volumetric analysis problem above.

Step 1: Find the number of moles of solute (sodium hydroxide) in the original solution.
Step 2: Find the number of moles of solute (sodium hydroxide) used in the titration.
Step 3: Find the molarity of the hydrochloric acid used in the titration.
Step 4: Obtain the volume of acid to be diluted.

The procedure contains just four steps to give an outline of how to solve the problem. There are no formulas or mathematical details; it is entirely qualitative. Following the generation of this procedure, the student then proceeded to solve the problem mathematically. Details, not included in this procedure such as volumes, molar masses and the formulas needed in each step, are filled in.

Less skilled problem solvers, lacking the knowledge and understanding of more skilled problem solvers, frequently omit the qualitative thinking when solving quantitative problems. Instead, they go directly from the initial representation to the mathematical representation by substituting values of variables into formulas which may lead to some solution. Such an approach seems to accompany the groping forwards strategy which less skilled students use when their knowledge base is inadequate. Even "top" students can rely on formula-driven solutions as McMillan and Swadener (1991) found when investigating the solving of quantitative electrostatics problems in physics.

Linkage

The construction of a qualitative representation results from links made between the elements of the problem description and the underlying knowledge base. Skilled problem solvers tend to have a good background knowledge of the topic which enables them to establish such links. Less skilled problem solvers do not make such links either because of a poorer background knowledge or, if this is present, they are unable to use the cues in a problem statement to make the necessary links (Osborne & Wittrock, 1983; Sumfleth, 1988). The specific effects of linkage and prior knowledge variables on problem solving were investigated with high school chemistry students in Australia and Singapore (Lee, 1985; Lee et al., 1996). The results showed that successful problem solving depended on both variables. Many students in the studies who scored high on a prior knowledge test were not good at solving problems as they were unable to make relevant links between the cues in the problem statements and their existing knowledge base.

Cognitive Load

The qualitative procedure above for solving the composite problem contains only four steps. The number of steps is a factor that must be considered in problem solving as the human information processing system has limited cognitive capacity. Current thinking is that working memory has a limit of four to five independent chunks of information (Halford, 1993). A small number of steps in a procedure ensures that working memory will not be overloaded with information. Cognitive load may be related to the strategy used, particularly that of means-ends analysis (Ayres, 1993; Sweller, 1988). In the study by Ayres (1993), Grade 7 students solved two-move geometry problems and it was found that when means-ends analysis was used, the number of errors at the subgoal stages increased. This was accounted for by an increase in the cognitive load at the subgoal stages due to the number of bits of information needed to be held in working memory. Studies in mathematics have further found that by setting goal-free problems, novice problem solvers did not tend to use means-ends analysis (which requires the goal to initiate the strategy) resulting in a decrease in errors and allowing solvers to more easily find the correct solution through the problem space (Ayres, 1993; Owen & Sweller, 1985).

The limited capacity of working memory can also be compensated for by increasing processing efficiency through the "chunking" of information during problem solving (Herron, 1990; White, 1988). Niaz (1995), in a study of quantitative problem solving in chemistry by university freshmen, found that some students used a two-step procedure to solve a problem, while others, as a result of greater expertise and conceptual understanding, had combined these steps into a single larger bit of information resulting in a one-step procedure for the same problem. Skilled problem solvers, by creating qualitative procedures with only a small number of steps ("chunks"), are able to greatly reduce the load placed on working memory. In addition, the well-rehearsed procedures for basic problems are probably stored in long-term memory as single chunks resulting in very little load on working memory when they are recalled to solve a problem. For composite problems, steps in a qualitative procedure will subsume these standard routine procedures for basic problems to further increase this efficiency. Thus Step 1 above in the procedure for solving the composite problem (i.e., "Find the number of moles of solute ... .") subsumes the standard procedure for calculating the number of moles of a solute. Similarly, Step 4 involving dilution subsumes the formula ($M_1V_1 = M_2V_2$) and the procedure for diluting a solution. Thus the four steps of the above qualitative procedure can be regarded as individual chunks that may subsume a large amount of information. Therefore, the generation by skilled problem solvers of high level, parsimonious qualitative procedures seems to automatically obviate difficulties that might occur due to limitations in cognitive processing capacity.

When less skilled students use the groping forwards strategy, the factor of cognitive load also seems to be less critical but for slightly different reasons. This is because students tend to execute each step numerically as it is generated removing the need to retain many formulas and steps in working memory. Thus the factor of cognitive load seems to be important in problem solving only if means-ends analysis is used and if the number of steps to be retained exceeds the 4 or 5 limit of working memory.

Knowledge and Understanding

Reference has already been made to the link between problem solving procedures and underlying knowledge. Science subjects, such as physics and chemistry, contain three levels of knowledge, namely, the macroscopic, the conceptual and the symbolic (cf. the description by Johnstone, 1991). The macroscopic level is a concrete level corresponding to observable objects, their properties and the terms used to describe them (from where the initial representation of a problem is derived). The conceptual level involves the concepts, theories and principles needed to explain what is observed at the macroscopic level (to generate the qualitative representation of a problem). The symbolic level deals with formulas and mathematical calculations (for the mathematical representation). Scientists and science teachers operate across all three levels of thought quite easily and switch from one mode of thinking to another without effort. Past research indicates that students have great difficulty with the conceptual level and develop many conceptions inconsistent with scientific ideas (e.g., Garnett, Garnett, & Hackling, 1995; Nakhleh, 1992). This level, of course, is outside their experience and can only be made accessible through the use of concrete models, analogies and graphics (Gabel, 1986; Johnstone, 1991). Difficulties related to volumetric analysis, the topic of concern in this study, include the concepts of mole and molarity (Duncan & Johnstone, 1973; Staver & Lumpe, 1995), the concept of volume (Enochs & Gabel, 1984), the particulate nature of matter (Griffiths & Preston, 1992; Novick & Nussbaum, 1978) and the balancing of chemical equations (Nurrenbern & Pickering, 1987; Yarroch, 1985).

In spite of conceptual difficulties, many students are still able to solve quantitative problems in science correctly (e.g., Gabel, Sherwood, & Enochs, 1984; Stewart, 1985), part of the reason for this being that students rely on algorithms, especially for basic or routine problems. In a comprehensive review of problem solving in chemistry, Gabel and Bunce (1994) found that students tended to rely primarily on algorithms to arrive at correct answers, rather than using their knowledge to create solutions. This is not, as might be expected, limited to less able problem solvers. Anamuah-Mensah (1986), for example, found that for solving basic titration problems in volumetric analysis at high school level, students of *all* achievement levels used algorithmic approaches.

Reasons put forward to account for the reliance by students on algorithms are that most teachers and general chemistry courses emphasize the application of algorithms to solve routine problems (Nurrenbern & Pickering, 1987) and that problems met in textbooks include procedures which can be used algorithmically to solve similar problems (Bodner, 1987). Another reason is that many instructors believe that the solving of routine algorithmic problems leads to conceptual understanding (Pickering, 1990; Sawrey, 1990) though a number of studies have shown this not to be the case (e.g., Gabel et al., 1984; Nurrenbern, 1979).

With new or unfamiliar problems, such as the composite volumetric analysis problem above, for which teachers or textbooks do not provide algorithms, students are forced to generate procedures creatively. Such problem solving requires the use of background knowledge and conceptual understanding. In solving the composite problem, skilled students seemed to make use of *both* understanding and algorithims. The qualitative procedures formed are the result of understanding and creativity. However, subsumed under the steps of these procedures are standard procedures corresponding to basic volumetric analysis problems which are called on and executed algorithmically when the numerical solution is obtained.

Many less skilled students have a poor conceptual understanding which prevents them from creating qualitative procedures. For basic problems, these students may be able to memorize given procedures without understanding and use them algorithmically for the future solving of identical problems. But for unfamiliar problems with unknown procedures, less skilled students resort to the formula-driven groping forwards strategy to assemble (rather than create) any numerical solution with whatever knowledge they may have. So even though these students may be able to pass conventional examinations using such strategies, they cannot really be regarded as genuine problem solvers if creativity is taken to be a criterion of a problem solver (e.g., Mettes, Pilot, Roossink, & Kramers-Pals, 1981; Gil-Perez & Martinez Terregrosa, 1983).

Directions for Practice

Skill in problem solving depends on the effective interaction of cognitive factors such as those discussed above. Why are some students better than others at problem solving and especially at generating procedures for solving more complex quantitative problems? A basic factor seems to be a good conceptual understanding of the topic. This understanding allows for good problem recognition and the setting up of a qualitative representation of the solution procedure with strategies that make efficient use of cognitive processing capacity. In order to improve problem solving skills, the standard approach is to look at the processes involved in skilled problem solving performance and then to derive instructional approaches that will assist practitioners. Based on the overview of quantitative problem solving presented in this paper, a number of ways by which this could be accomplished are discussed below.

1. Teach for Conceptual Understanding

A conceptual understanding of the topic must be obtained before students are given problems to solve, rather than trying to get this understanding by means of problem solving. As many of the concepts are abstract in nature, care must be taken that they are introduced concretely. For example, most students have difficulties with concepts in volumetric analysis. Consider just one example, that of molarity. This is often poorly understood with students memorizing the definition for molarity and the formula: molarity = moles/volume. The concept of molarity is related to that of concentration which can be introduced concretely using *colored* substances such as concentrated and dilute orange juice. To show the relationship between concentration, amount of solute and volume, colored solutions should again be used (rather than using colorless acid and alkali solutions). For example, by adding 1 spoonful of orange potassium dichromate crystals to a unit volume of water (say half a beaker full) and 2 spoonfuls to another equal volume, students can readily see from the intensity of the colour how amount of solute affects concentration. Similarly, by keeping the number of spoonfuls of solid constant and varying the volume, students can better comprehend how concentration is affected by volume. This leads easily into concentration as amount of solute/volume of solution for which the chemical terms of molarity (for concentration) and moles (for amount) can then be introduced. Through this concrete approach the concept of molarity and the formula molarity = moles/volume are better understood and "make sense" to students. But even with the best instruction, misconceptions can still arise and teachers should continually monitor students' understanding and correct any misconceptions that are detected.

2. Encourage Qualitative Thinking

Skilled students, in contrast to the less skilled students, have a good qualitative understanding of problems. Rather than just giving numerical procedures which may be memorized and used without understanding, allowing students the opportunity to *think aloud* while solving a problem and to derive *qualitative non-mathematical* procedures for problems could facilitate qualitative understanding. This would include talking about key words to identify problems and the decomposition of composite problems into component parts. Qualitative discussions could be carried out while problems are solved on the chalkboard and also by getting students to work together, say in pairs, while solving problems with students being asked to derive general procedures rather than mathematical solutions. Thinking aloud would also help teachers and students to identify misconceptions or reveal areas of knowledge not clearly understood but which are needed to solve problems.

3. Set Goal-Free Problems

When introducing problems, include goal-free problem statements so that students have to work out what can be deduced from the information. For example, instead of giving the following complete problem,

> In a titration, a student finds that 50 cm^3 of a 0.8 M sodium hydroxide solution completely neutralizes 20 cm^3 of a hydrochloric acid solution. What is the concentration of the hydrochloric acid?

the final sentence containing the goal statement is omitted to leave:

> In a titration, a student finds that 50 cm^3 of a 0.8 M sodium hydroxide solution completely neutralizes 20 cm^3 of a hydrochloric acid solution.

Students should be encouraged to think about the problem and deduce anything that can be obtained from it. This could have several beneficial effects. It should facilitate genuine problem solving by requiring the use of understanding in the creation of procedures (as well as revealing any lack of understanding), it should help students to identify the basic problems when solving composite problems (as these do not contain subgoal statements) and it should allow for the use of the working forwards strategy favored by skilled students. The solving of goal-free problems could beneficially be combined with thinking aloud as in point 2.

4. Practice with Basic Problems

Once students have derived *and* understood procedures for basic problems, they should be given plenty of practice to the extent that the problems can be solved algorithmically. Then, for future problem solving, these procedures can be accessed from long-term memory as individual chunks thus reducing cognitive load. This is especially important when longer, more complex problems are solved which contain the basic procedures as components of the overall solution.

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